Avoiding Economic Crises via the Stochastic Money Supply

Falip. O. Lor Ph.D[1],

[This version: 1.4.09]

Abstract

We generalise the arguments of Minsky to propose a stochastic differential equation (SDE) using GBM for modelling the domestic money supply and its effect on GDP via the velocity of money *v*. Using Prospect Theory's positive convexity of investors' risk preferences under framing losses, and assuming the scaling behavior of the tails of asset returns, we can show that positive feedback loop leading to bubbles in the economies can be dampable if central banks will take action to limit the volatility of volatility of *v*.

We solve our equation in discreet time via simulation involving cellular automata. Extending the stochastic behavior of money supply to allow for Levi-Khinchine processes, we have used Malliavin calculus to demonstrate that our results still hold true in aggregate.

Finally, we propose a new risk measure, CVaR-*V*, measuring the bubble risk in a domestic economy. Assuming the validity of the Black-Litterman model, we extend our results to economies of several coupled countries for all logarithmic utility functions[2].

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1. Introduction

In the recent literature on macroeconomic activity under rational expectations, it has been argued that systematic monetary policy is ineffective with regard to real economic variables. In other words, changes in money supply rules leave the distributions of real variables unaffected. We excerpt here from Minsky's classic manuscript on this topic as an inspiration towards approaching this probleme more rigourously.

> In accordance with the conventional formulations of stabilization policy objectives, we assume that the monetary authority seeks to minimize the deviations of national product from its "natural rate" (at which no errors in product price expectations occur). To set the stage, we first show that systematic monetary policy is irrelevant to these objectives whenever all the random disturbance terms enter the model additively. Second, we examine how multiplicative disturbance terms provide scope for stabilization policy through systematic monetary rules. Lastly, we indicate that the existence of multiplicative disturbances does not guarantee the effective-ness of systematic monetary policy.

The heterodox approach of Minsky[1] to central bank regulation has indeed recently become popular again. In this paper we generalize his approach to the case of stochastic money supplies.

The work proceeds as follows.

Section 1. Introduction
Section 2. Matheamatical Foundations
Section 3. The Monetary Velocity in the Keynesian Model.
Section 4. An SDE for the Central Banks Money Supply.

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Section 5. Solving
Section 5. Extensions.

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The policy-ineffectiveness argument emerges straightforwardly from Equations (1) - (4). By Equation (2) and (3),

(5) $$P = \overline{M} + u^m + u^D$$

and thus, by Equation (4),

(6) $$P_e = \overline{M}$$

Substituting Equations (5) and (6) into (1), we find

(7) $$Q = a \left[u^m + u^D \right] + u^s$$

From this equation it is evident that the systematic money supply

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Remarks: - D is well-dened. In particular for F = W (h) =
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ACknowledgements

Ωε πριχε α σιμπλε ονε–ψεαρ Ευροπεαν ωαρραντ ον α στοχκ τηατ ισ τραδινγ ατ ∃100, ωιτη α ϖολατιλιτψ οφ 20% περ ψεαρ ανδ αν αννυαλ διϖιδενδ ψιελδ οφ 1%. Τηε ιντερεστ ρατε ισ 5% αννυαλλψ χομπουνδεδ. Ωε στυδψ τηε διφφερενχε ιν πριχινγ υσινγ βοτη τηε οπτιονσ αππροαχη ανδ ουρ αππροαχη, φορ α ϖαριετψ οφ στρικεσ ανδ διλυτιονσ.

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