

A Poor Man's Guide To Quantitative Finance

Emanuel Derman
October 2002

Email: emanuel@ederman.com

Web: www.ederman.com

PoorMansGuideToQF.fm
September 30, 2002

Summary

Quantitative finance employs much of the language and techniques of physics.
How similar are the two fields? What principles do you use in the practitioner world?

- Models in Physics
- Models in Finance
- Two Preambles
- The One Commandment of Financial Modeling
- Using the Law
- How Do You Tell When a Model is Right?

Models in Physics

- Fundamental Models or Theories

Fundamental models attribute effects to deep dynamical causes.

Kepler's laws of planetary motion — not quite a theory:

- planets move about the sun in elliptical orbits;
- the line from the sun to the planet sweeps out equal areas in equal times; and
- $(\text{period})^2 \sim (\text{radius})^3$.

Nevertheless, the laws do provide profound insight.

Newton adds dynamics, provides a fundamental theory.

- Phenomenological Models

A toy or analogy to help visualize something unobservable.

As-if models.

- Liquid drop model of nucleus;

Calibrate to known phenomena, then use it to predict the unknown.

Models in Finance

There is no fundamental theory in finance. There are no laws.
That's why many of the textbooks are so mathematically rigorous.

- Phenomenological Models

Models in finance are used to *turn opinions into prices*, and prices into implied opinions. (Cf: fruit salad).

Most financial models assume that investors make *simple rational assessments*, based on a few *intuitively understandable variables* which represent the market's *opinion of the future*: dividends, interest rates, volatilities, correlations, default rates, etc. Models are causal and perturbative.

The *implied values* of these variables are determined by calibrating/renormalizing the model to liquid market prices.

Prices are often non-linear in these variables, while intuition about their value is more reliably linear. One uses the models to interpolate smoothly from known to unknown.

- Statistical Models

Not models in the physics sense. Physicists use statistics to test theories. Economists use it to find relationships and so make theories. Mostly regression without explicit dynamics, and therefore non-perturbative. Useful when you *have* to have some estimate.

Two Preambles

- Preamble 1.

It's common to imagine that quants on Wall St. spend their time predicting the future. They rarely do. Most do quantitative *haute couture*, using models to value and price custom-tailored securities from off-the-shelf products.

- Preamble 2.

Pure arbitrage is simultaneously buying at one price and selling at another. It's amazingly rare.

What's more sloppily called arbitrage is finding a discrepancy between a model price and a market price, and acting on it.

The One Commandment of Financial Modeling

God's laws while standing on one leg:

Do not do unto others as you would not have them do unto you. All the rest is commentary. Go and learn.

The law of financial modeling while standing on one leg:

If you want to know the value of a security, use the price of another security that's similar to it. All the rest is strategy. Go and build a model.

Financial economists call their version *the law of one price*:

Any two securities with identical future payoffs, no matter how the future turns out, should have identical current prices.

Using the Law of One Price

To value a *target* security, find some other *replicating portfolio* of *liquid securities* with the same future payoffs in all states of the world.

The value of the target is the value of the replicating portfolio.

- The Role of Models

Models are used to prove the identity of the future payoffs:

1. Since the future is uncertain, you must model that uncertainty by specifying the range and probability of future scenarios for the prices of all relevant securities.
2. You need a strategy for creating a replicating portfolio that, in each of these future scenarios, will have identical payoffs to those of the target security.

Replication can be *static* or *dynamic*.

1. Valuing (more or less) riskless bonds

Find a bond with similar credit. Parametrize its present value by means of discount factors:

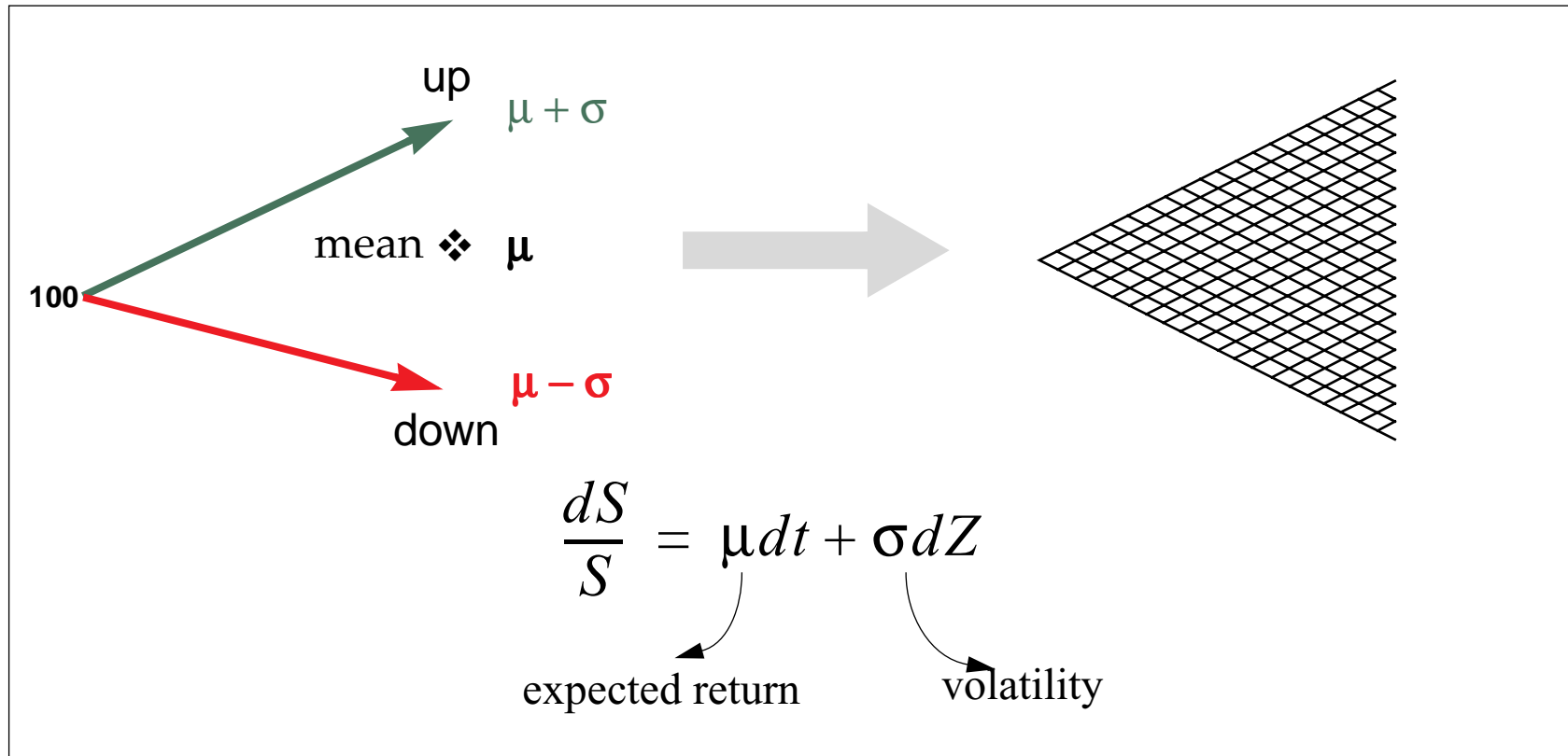
$$P(t) = \exp[-r_t t] \quad \text{discount factor}$$

Rate of return r is the appropriate conceptual variable for comparing investments, just as velocity is the right variable to compare modes of transport.

Forward rates are the future rates of growth f you can lock in by buying and selling today. A very important way of thinking.

2. Model risky securities via uncertain growth.

Simple model: assume probability of an up or down move is 1/2.



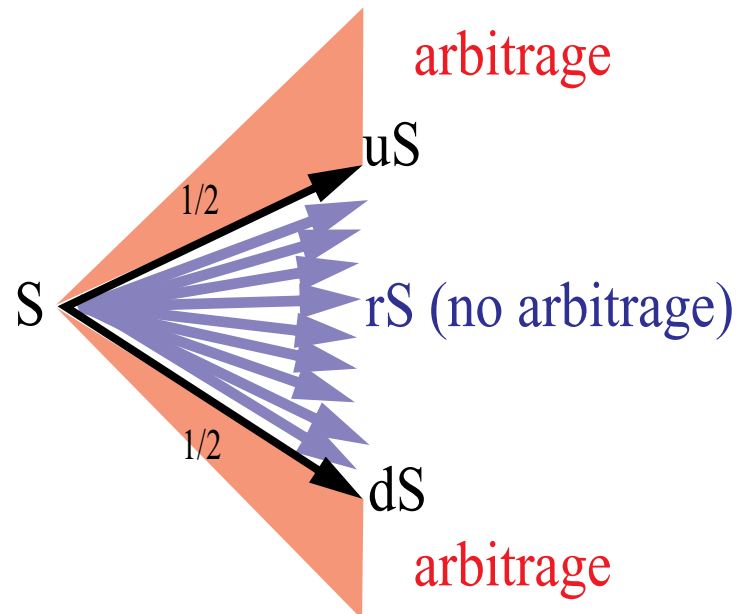
One *can* build more sophisticated models of uncertainty.

- Stochastic calculus: dZ is a Brownian motion: mean zero, standard deviation \sqrt{t}

$$dS \sim dt \quad dS^2 \sim dt$$

3. Avoid Riskless Arbitrage

A risky portfolio *must* bear risk.



Possible returns must bracket the riskless rate.

Therefore, the riskless return r is a convex combination of u and d .

$$pu + (1 - p)d = r$$

$p \equiv (1/2 + \lambda)$ is a probability measure *for each stock*.

This is why the probabilistic thread runs throughout modern finance.

λ is the *risk premium* of the stock.

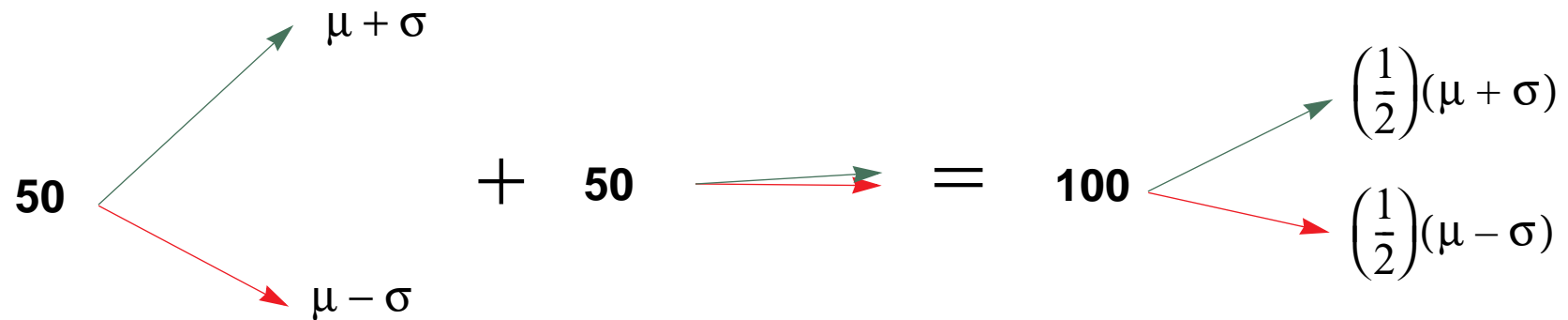
Remember: No arbitrage is a constraint on *how we model the world*.

4. The Fundamental Question: Risk vs. Return?

- Use a diluted law of one price:

Two portfolios with the same perceived instantaneous risk should have the same expected return.

You can make a low-volatility portfolio out of a high-volatility portfolio and cash.:



Half the risk, half the return.

$$\frac{\mu - r}{\sigma} = \lambda$$

Excess return per unit of risk is the same for all stocks, and is equal to the risk premium λ .

- More risk, more return.
- What is the value of λ ?

5. Risk Reduction By Diversification

- The removal of risk by the law of large numbers.

If

you can buy a very large portfolio of stocks,
and their returns are uncorrelated

Then

asymptotically the portfolio volatility $\sigma \rightarrow 0$
and so its return $\mu \rightarrow r$.

Therefore the return of each stock must be the riskless rate, and therefore

$\lambda = 0$ Zero risk premium, expect riskless growth

- You are not paid to take on diversifiable risk.

6. Risk Reduction By Hedging

- The removal of risk by cancellation of common factors.
- Example: all stocks are correlated with the market M . You can remove this component of risk from any stock by combining it with a short position in the market.
- $\mu_i - \beta\mu_M$ carries no market risk, only residual risk, where $\beta = \rho_{iM} \frac{\sigma_i}{\sigma_M}$

You can diversify over all stocks. Since you are not paid to take on diversifiable risk,

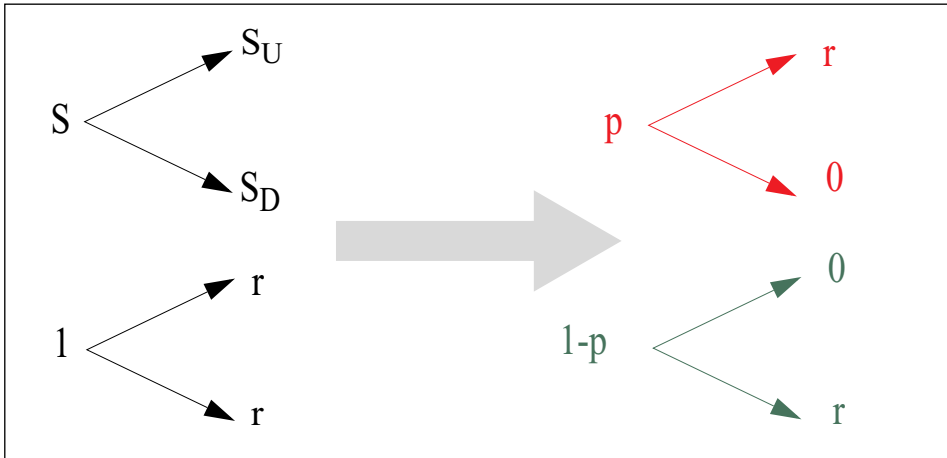
$$(\mu_S - r) = \beta_{SM}(\mu_M - r) \quad \text{CapM}$$

The expected return of a stock is proportional to the market's return times its co-movement with the market. Similarly for more correlated factors, get the APT results.

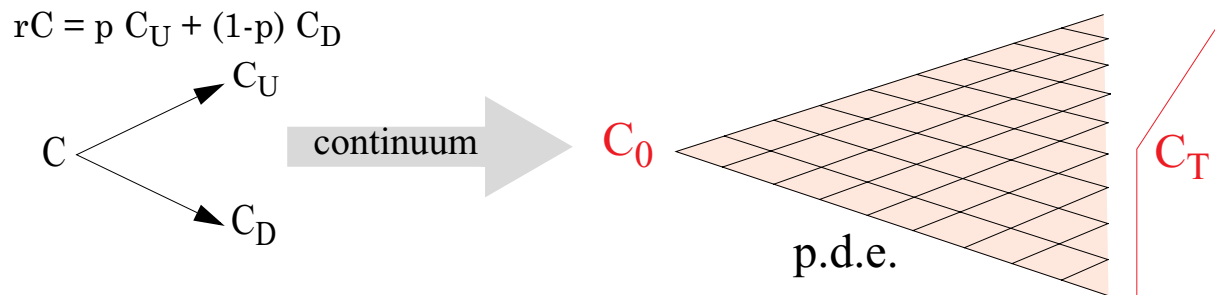
7. Options are not independent securities

Options (derivatives) have pay-offs which are curved (non-linear) functions of the stock price.

A stock and a bond can be decomposed into Arrow-Debreu securities p and $1-p$ that span the price space for a short time Δt :



With these one-state securities p and $1-p$, you can dynamically replicate the payoff of the non-linear option $C(S)$ at each instant:



Since you can replicate, you don't care about path of stock, only its volatility.
Options traders bet on volatility.

8. Extensions of replication (the past 25 years)

- Extension of the Black-Scholes-Merton replication method of pricing derivatives on currencies, commodities, interest-rate-sensitive securities, mortgages, credit-derivatives, etc.

Strategy:

- build a realistic model of the stochastic behavior of the underlying security;
 - Calibrate it to the current prices of the underlying security;
 - Figure out how to replicate the derivative security;
 - Find the value of the derivative by backward induction or Monte Carlo simulation of the replication process.
- Traders have become more analytical as they realize that they are trading volatility, and develop both an understanding and a feel for the model.
 - Markets are able to *estimate* the value of exotic and hybrid options.
 - Perturbation of the idealized Black-Scholes model to take account of the real world and *behavioral and perceptual issues*:
 - illiquidity, transactions cost, noncontinuous trading, skew, non-normal distributions, market participants behavior, etc.

9. One last trick: change of numeraire

- The most ubiquitous trick in finance.

The value of an instrument should be independent of the currency you choose to model it in.

The currency can be a dollar, a yen, the value of an IBM share, the value of anything tradeable.

Choosing an currency or numeraire allows you to make simpler models for complex products.

It s a bit like choosing looking at P/E, choosing earning as the numeraire with which to compare different stocks. Choosing a numeraire is a kind of model too.

How do you tell when a model is right?

- Trading with a model is not the simple procedure academics imagine. Intelligent traders iterate between imagination and model in a sophisticated way.
- Traders use models as gedanken experiments, as theoretical laboratories or parallel thought universes for testing cause and effect. Models provide a common language to communicate opinions and values. No single one is right.
- A good model is easily embraceable because it incorporates concepts that allow you to stress-test the world in your imagination. It asks just enough of you, but not too much.
- Building trading systems that make it easy to use models is as hard as building the models themselves.
- Fischer Black (1986):

In the end, a theory is accepted not because it is confirmed by conventional empirical tests, but because researchers persuade one another that the theory is correct and relevant.

Models in the social sciences are only models, toy-like descriptions of idealized worlds that can only approximate the hurly-burly, chaotic and unpredictable world of finance and people and markets.